Available online at www.sciencedirect.com
SCIENCE DIRECT®
PERGAMON
International Journal of Heat and Mass Transfer 46 (2003) 1675–1678
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

# Laminar free-convection in glycerol with variable physical properties adjacent to a vertical plate with uniform heat flux

A. Pantokratoras *

*School of Engineering, Democritus University of Thrace, 67100 Xanthi, Greece*

Received 31 March 2002

**Abstract**

The steady laminar boundary layer flow of glycerol along a vertical stationary plate with uniform heat flux is studied in this paper. The density, thermal conductivity and heat capacity of this liquid are linear functions of temperature but dynamic viscosity is a strong, almost exponential, function of temperature. The results are obtained with the numerical solution of the boundary layer equations. Both upward flow (plate heating) and downward flow (plate cooling) is considered. The variation of $\mu$ with temperature has significant influence on wall heat transfer and much stronger influence on wall shear stress. It was also found that the similarity exponent, which is equal to 0.20 for the classical problem with constant properties, is lower than 0.20 in the upward flow and higher than 0.20 in the downward flow.


*Keywords:* Heated vertical plate; Free convection; Uniform heat flux; Glycerol; Similarity

## 1. Introduction

It is known that many fluid properties (dynamic viscosity, thermal conductivity and heat capacity) are functions of temperature. For most fluids viscosity is more sensitive to temperature variations than heat capacity and thermal conductivity. Liquids can undergo strong viscosity variations which are usually well described by exponential laws. On the other hand, heat capacity and thermal conductivity for gases and liquids are well represented by linear functions of temperature.

The earliest known theoretical treatment of free convection along a vertical plate with uniform heat flux is the analysis of Sparrow and Gregg [10]. There are numerous subsequent similar investigations in the literature. The reader can find many relevant works in a very recent paper by Aydin and Guessous [1]. Reviews for several natural convection flows with variable physical properties can also be found in the works of Carey and Mollendorf [2] and Kakac et al. [6]. In the present paper we focus on glycerol for the following reasons. The density, thermal conductivity and heat capacity of this liquid are almost linear functions of temperature but dynamic viscosity is a strong function of temperature. These functions are given by Cuckovic-Dzodzo et al. [3] for temperatures between 20 and 60 °C. For the temperature range between 60 and 20 °C the glycerol Prandtl number varies from 825 to 10 785. The glycerol viscosity is about 1000 times that of water at 20 °C. Although glycerol has been used as working fluid in some previous works [3,5,9], to the best of the author's knowledge, glycerol has not been used in the classical problem of natural convection along a vertical plate with uniform heat flux.

The boundary layer equations with variable fluid properties were solved directly, without any transformation, by a method described by Patankar [8]. The functions given by Cuckovic-Dzodzo et al. [3] have been used for the calculation of fluid properties. The finite difference method is used with primitive coordinates $x$, $y$ and a space marching procedure is used in $x$ direction with an expanding grid. The accuracy of the method was

* Tel.: +30-541-079618.
*E-mail address:* apantokr@civil.duth.gr (A. Pantokratoras).

0017-9310/03/$ - see front matter 
PII: S0017-9310(02)00439-8

**Nomenclature**

| | |
|---|---|
| $f$ | dimensionless stream function |
| $g$ | gravitational acceleration |
| $Gr_x$ | local Grashof number, $Gr_x = (gx^3/\nu^2)(\rho_a - \rho_o/\rho_a)$ |
| $k$ | thermal conductivity |
| $Nu_x$ | local Nusselt number, $Nu_x = qx/k(T_o - T_a)$ |
| $Pr$ | Prandtl number, $Pr = \nu/\alpha$ |
| $q$ | surface heat flux |
| $Ra^*$ | modified Rayleigh number, $Ra^* = g\beta qx^4/\nu\alpha k$ |
| $T$ | glycerol temperature |
| $u$ | vertical velocity |
| $v$ | horizontal velocity |
| $x$ | vertical coordinate |
| $y$ | horizontal coordinate |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\beta$ | thermal expansion coefficient of glycerol |
| $\eta$ | similarity variable, $\eta = (y/x)(Gr_x/4)^{1/4}$ |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity |
| $\rho$ | glycerol density |
| $\theta$ | dimensional temperature, $\theta = T - T_a/(T_o - T_a)$ |

*Subscripts*

| | |
|---|---|
| a | ambient |
| o | plate |
| f | film |

tested comparing the results with those of the classical free convection problem with constant properties [4]. The comparison was satisfactory. More information about the equations and the solution procedure may be found in Pantokratoras [7]. The boundary conditions were as follows:

$$\text{at } y = 0: \quad u = v = 0, \quad -k\left[\frac{\partial T}{\partial y}\right] = q \tag{1}$$

$$\text{as } y \to \infty \quad u = 0, \quad T = T_a \tag{2}$$

where $q$ is the heat flux at the plate and $T_a$ is the ambient temperature.

## 2. Results and discussion

In the similarity method commonly used in free convection over vertical surfaces, the following functions and variables are used. The nondimensional stream function $f(n)$, the similarity variable $n$, the local Grashof number $Gr_x$ and the nondimensional velocity $f'$. These quantities are well known and can be found in the literature (see for example [7]). The most important quantities for this problem are the wall heat transfer and the wall shear stress defined as

$$\theta'(0) = \frac{x}{T_o - T_a}\left(\frac{Gr_x}{4}\right)^{-1/4}\left[\frac{\partial T}{\partial y}\right]_{y=0} \tag{3}$$

$$f''(0) = \frac{\rho_o x^2}{\mu_o\sqrt{2}}(Gr_x)^{-3/4}\left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{4}$$

where $T_o$ and $T_a$ are the plate and ambient temperatures and $\rho_0$ and $\mu_o$ are the the glycerol density and dynamic viscosity at the wall.

It is known from the literature that in the classical free convection with uniform surface heat flux the plate temperature increases along the plate according to the law

$$\Delta T = T_o - T_a = Nx^n \tag{5}$$

where the similarity exponent $n$ is equal to $1/5$. In the present work results were produced for four ambient temperatures ($T_a = 20$, 30, 40 and 50 °C). In each numerical experiment the plate temperature varied from the ambient temperature to 60 °C ($\Delta T = 40$, 30, 20 and 10 °C). In addition to the usual problem of the upward moving fluid, results have been produced for downward flow. This can be achieved by cooling the plate instead of heating it ($q$ was taken negative in Eq. (1)). In this case the plate temperature decreases along the plate. For the downward flow results were produced again for four ambient temperatures ($T_a = 60$, 50, 40 and 30 °C). In each numerical experiment now the plate temperature varied from the ambient temperature to 20 °C ($\Delta T = 40$, 30, 20 and 10 °C).

In each numerical experiment the local Grashof and the local Prandtl number have been considered variable along the flow. The solution procedure starts from the plate leading edge and marches in the vertical direction. At every downstream position we calculated the plate temperature and then the local Grashof and the local Prandtl number have been calculated at film temperature $(T_o + T_a)/2$ which also changes along the plate. The correspondence between the Prandtl number and the glycerol temperature is as follows:

| $T$ (°C) | 60 | 50 | 40 | 30 | 20 |
|---|---|---|---|---|---|
| $Pr$ | 825 | 1263 | 2310 | 4696 | 10 785 |

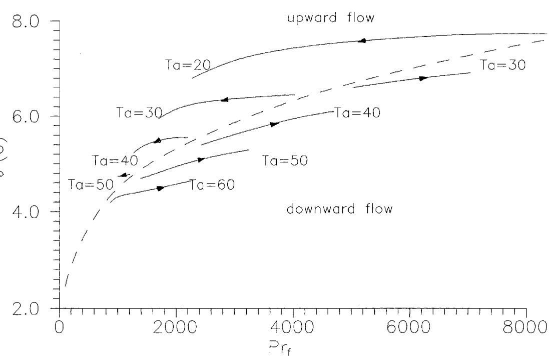


Fig. 1. Wall heat transfer as function of film *Pr* number for different ambient temperatures. Dashed line corresponds to constant properties. The curves above the dashed line correspond to upward flow and those below the dashed line to downward flow. Arrows show increasing $\Delta T$.

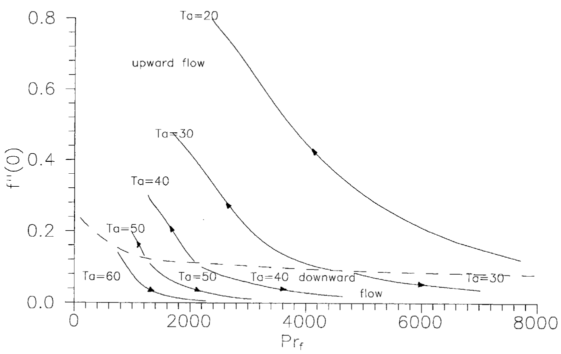


Fig. 2. Wall shear stress as function of film *Pr* number for different ambient temperatures. Dashed line corresponds to constant properties. The curves above the dashed line correspond to upward flow and those below the dashed line to downward flow. Arrows show increasing $\Delta T$.

In Fig. 1 the wall heat transfer $-\theta'(0)$ is shown as function of the film Prandtl number $Pr_f$ for different ambient temperatures. The dashed line corresponds to heat transfer of the classical problem with constant properties. Values of $-\theta'(0)$ for the classical problem and high Prandtl numbers do not exist in the literature. In the work of Gebhart [4] a value of 2.4584 is given for $Pr = 100$. Values for other *Pr* numbers have been calculated by the present method. The curves above the dashed line correspond to the upward glycerol flow and those below the dashed line to downward flow. Arrows show increasing $\Delta T$. From this figure it is seen that as $\Delta T$ increases the wall heat transfer $-\theta'(0)$ decreases in the upward flow and increases in the downward flow. As $\Delta T$ increases the departure from the constant properties curve increases, too. From this figure it is also seen that the heat transfer with variable properties is higher than that of constant properties in the upward flow and lower than that of constant properties in the downward flow.

In Fig. 2 the wall shear stress $f''(0)$ is shown for the same conditions of Fig. 1. Now the dashed line corresponds to wall shear stress of the classical problem of free convection with constant properties. Unfortunately there are no arithmetic results of $f''(0)$ for the classical problem and high *Pr* numbers. In the work of Gebhart [4] for $Pr = 100$ a value of 0.2367 is given. Values for other *Pr* numbers have been calculated by the present method and are shown by the dashed line in Fig. 2. From this figure it is seen that as $\Delta T$ increases the wall shear stress increases in the upward flow and decreases in the downward flow. As $\Delta T$ increases the departure from the dashed line increases, too. The effect of temperature dependent viscosity on wall shear stress is much stronger than that of the wall heat transfer. Again the shear stress with variable properties is higher than that of constant properties in the upward flow and lower than that of constant properties in the downward flow.

Another interesting quantity calculated in the present work is the value of the exponent $n$ in Eq. (5). In the classical problem with constant properties the value of $n$ is 0.20. In the present case where the dynamic viscosity is a strong function of temperature the value of $n$ deviates from the above value. In the following table the calculated values of $n$ for different temperature ranges are given.

| | | | | |
|---|---|---|---|---|
| *Upward flow* | | | | |
| Temperature range (°C) | $20 \Rightarrow 60$ | $30 \Rightarrow 60$ | $40 \Rightarrow 60$ | $50 \Rightarrow 60$ |
| $n$ | 0.1580 | 0.1717 | 0.1828 | 0.2000 |
| *Downward flow* | | | | |
| Temperature range (°C) | $60 \Rightarrow 20$ | $50 \Rightarrow 20$ | $40 \Rightarrow 20$ | $30 \Rightarrow 20$ |
| $n$ | 0.2685 | 0.2765 | 0.2658 | 0.2406 |

From the above table it is seen that the exponent $n$ is lower than 0.20 in the upward flow and higher than 0.20 in the downward flow. It should be noted here that in the above temperature ranges the temperature variation along the plate is represented adequately by a power law but as the viscosity nonlinearity increases the temperature variation departs from the power law. We performed some calculations in the temperature range between 0 and 20 °C, where the glycerol viscosity is much more nonlinear, and we found that the temperature variation can be simulated only by a polynomial or logarithmic law.

Another interesting quantity in heat transfer problems is the local Nusselt number defined as

$$Nu_x = \frac{qx}{k(T_o - T_a)} \tag{6}$$

The most recent correlation of Nusselt number for free convection along a vertical plate with constant heat flux is that by Aydin and Guessous [1]. They proposed the following equation which correlates the local Nusselt number, the local Prandtl number and the modified Rayleigh number

$$Nu_x = C_1 \left[ \frac{Ra^* Pr}{0.67 + Pr} \right]^m \tag{7}$$

where the exponent $m$ is again 0.20 and the constant $C_1 = 0.630$. These values are valid for fluids with constant properties. In the present work the local Nusselt number has been calculated in each numerical experiment from Eq. (6) and the values have been compared with the results of Eq. (7). It is reminded here that the local modified Rayleigh number and the local Prandtl number, used in Eq. (7), have been considered again variable along the flow. At every downstream position we calculated the plate temperature and then the two numbers have been calculated at film temperature. The biggest differences between our results and those by Aydin and Guessous [1] were 13% (temperature ranges $20 \Rightarrow 60$ and $60 \Rightarrow 20$). These differences are reasonable because Eq. (7) concerns fluids with constant properties. These differences are not so significant taking into account the strong dependence of viscosity on temperature. This happens because the local Rayleigh number and the local Prandtl number have been considered variable along the flow. In this way the viscosity nonlinearity is incorporated in Eq. (7) step by step and thus its influence becomes minimum. In order to minimize the above differences we tried to adjust the exponent $m$ and the constant $C_1$ to our results. In the following table the calculated values of $m$ and $C_1$ for different temperature ranges are given.

| | | | | |
|---|---|---|---|---|
| *Upward flow* | | | | |
| Temperature range (°C) | $20 \Rightarrow 60$ | $30 \Rightarrow 60$ | $40 \Rightarrow 60$ | $50 \Rightarrow 60$ |
| $m$ | 0.2053 | 0.2026 | 0.2012 | 0.2000 |
| $C_1$ | 0.634 | 0.659 | 0.664 | 0.667 |
| *Downward flow* | | | | |
| Temperature range (°C) | $60 \Rightarrow 20$ | $50 \Rightarrow 20$ | $40 \Rightarrow 20$ | $30 \Rightarrow 20$ |
| $m$ | 0.1938 | 0.1903 | 0.1911 | 0.1923 |
| $C_1$ | 0.705 | 0.772 | 0.768 | 0.772 |

From the above table it is seen that the exponent $m$ is higher than 0.20 in the upward flow and lower than 0.20 in the downward flow. The differences between our results and those by Eq. (7) with the above modified $C_1$ and $m$ values are below 1% for all temperature ranges.

## References


[1] O. Aydin, L. Guessous, Fundamental correlations for laminar and turbulent free convection from a uniformly heated vertical plate, Int. J. Heat Mass Transfer 44 (2001) 4605–4611.

[2] V.P. Carey, J.C. Mollendorf, Variable viscosity effects in several natural convection flows, Int. J. Heat Mass Transfer 23 (1980) 95–109.

[3] D.M. Cuckovic-Dzodzo, M.B. Dzodzo, M.D. Pavlovic, Laminar natural convection in a fully partitioned enclosure containing fluid with nonlinear thermophysical properties, Int. J. Heat Fluid Flow 20 (1999) 614–623.

[4] B. Gebhart, Similarity Solutions for Laminar External Boundary Region Flows, Natural Convection, Fundamentals and Applications, Hemisphere Publishing Corporation, Washington, 1985.

[5] C.G. Jeevaraj, J.C. Patterson, Experimental study of transient natural convection of glycerol–water mixtures in a side heated cavity, Int. J. Heat Mass Transfer 35 (1992) 1573–1587.

[6] S. Kakac, O.E. Atesoglu, Y. Yener, The effects of the temperature-dependent fluid properties on natural convection-summary and review, natural convection, fundamentals and applications, Hemisphere Publishing Corporation, Washington, 1985.

[7] A. Pantokratoras, Laminar free-convection over a vertical isothermal plate with uniform blowing or suction in water with variable physical properties, Int. J. Heat Mass Transfer 45 (2002) 963–977.

[8] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, McGraw-Hill Book Company, New York, 1980.

[9] J.F.T. Pittman, J.F. Richardson, C.P. Sherrard, An experimental study of heat transfer by laminar natural convection between an electrically-heated vertical plate and both Newtonian and non-Newtonian fluids, Int. J. Heat Mass Transfer 42 (1999) 657–671.

[10] E.M. Sparrow, J.L. Gregg, Laminar free convection from a vertical plate with uniform surface heat flux, Trans. ASME 78 (1956) 435–440.